When I review the first example of your lecture today, I come up with another example:

$$1+2+3+4+\cdots+n+\cdots-\left(2+4+6+8+\cdots+2n+\cdots\right) \tag{1}$$

And we can have two ways to view this, as we did in the case of $\sum_1^\infty \frac{1}{n(2n+1)}$.

$$\sum_1^\infty \left[(2n-1)+2n-2n\right]=+\infty \quad \text{and} \quad \sum_1^\infty n-2\sum_1^\infty n=-\infty\,. \tag{2}$$

So I think maybe this is all about an accurate expression and (1) is one with ambiguity.

Then I come back to the example in the lecture, and I guess, actually,

$$\sum_1^\infty \frac{1}{n(2n+1)}=\sum_1^\infty\left(\frac{1}{n}-\frac{2}{2n+1}\right)\neq\sum_1^\infty\frac{1}{n}-2\sum_1^\infty\frac{1}{2n+1}\,. \tag{3}$$

Because they are talking about different things, just like two understandings of Eq.(1). When the upper limit is finite, everything is clear, but when it goes to infinity, it will lead to some misunderstanding. My first questions are:

A) Is my understanding about this correct?

B) If the inequality above is correct, we have to be extremely careful when separating a series in the Σ (or maybe Π as well), so what is the general rule for us to separate a certain series?

According to what you said in the class, the distance between two terms should not grow larger and larger. But when it goes to like:

$$\sum_1^\infty\left(\frac{1}{n}-\frac{1}{n^2}\right) \quad \text{or} \quad \sum_1^\infty\left(\frac{1}{n}-\frac{1}{\ln n}\right). \tag{4}$$

It is alright just to separate into two series?