Problem 5) a) Differentiating $z(r,\phi,t)$ with respect to time and setting $t=0$, we find

$$\left.\frac{\partial z(r,\phi,t)}{\partial t}\right|_{t=0} = -\sum_{n=1}^{\infty} c_n \omega_n J_0(r_{0n} r/R)\sin(\omega_n t)_{t=0} = 0.$$

The present problem is a special case of Problem 82, from which we now borrow the following results.

b) The vibration frequency ω_n is denoted by C in Problem 82. Therefore, $\omega_n = v r_{0n}/R$.

c) The initial condition is obtained by setting $t=0$ in the general expression of the vibration amplitude, that is,

$$h(r) = \sum_{n=1}^{\infty} c_n J_0(r_{0n} r/R).$$

To determine the coefficients c_n, we take advantage of the orthogonality of the functions $J_0(r_{0n} r/R)$ over the interval $[0, R]$. In accordance with the Sturm-Liouville theory, the Bessel functions appearing in the above series are orthogonal with a weighting function $r(x)=x$. We thus write

$$\int_0^R r h(r) J_0(r_{0m} r/R)\,\mathrm{d}r = \sum_{n=1}^{\infty} c_n \int_0^R r J_0(r_{0m} r/R) J_0(r_{0n} r/R)\,\mathrm{d}r = c_m \int_0^R r J_0^{\,2}(r_{0m} r/R)\,\mathrm{d}r.$$

The coefficient c_m is readily found to be

$$c_m = \frac{\int_0^R r h(r) J_0(r_{0m} r/R)\,\mathrm{d}r}{\int_0^R r J_0^{\,2}(r_{0m} r/R)\,\mathrm{d}r}.$$