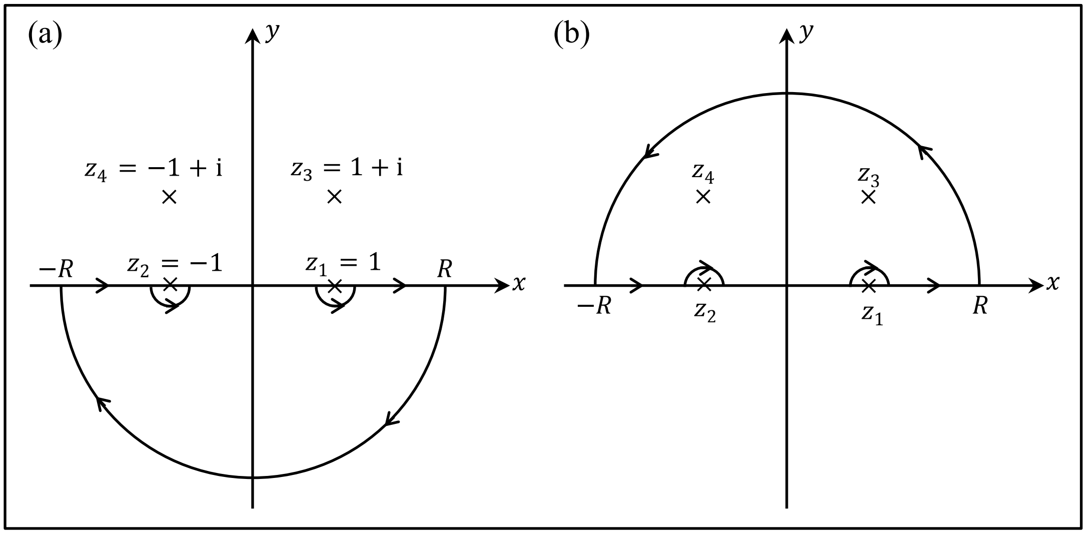
**Problem 3**) The poles of the integrand in the complex $z$-plane are readily found to be

$$(z^2-1)(z^2-2\mathrm{i}z-2)=0 \quad \rightarrow \quad z_{1,2}=\pm 1 \quad \text{and} \quad z_{3,4}=\mathrm{i}\pm\sqrt{\mathrm{i}^2+2}=\pm 1+\mathrm{i}.$$

Both contours of integration depicted in Figs.(a) and (b) are acceptable, since, in each case, the integral over the large semi-circle goes to zero when $R\rightarrow\infty$.




In Fig.(a) the contour is closed in the lower-half of the $z$-plane. The closed contour does *not* contain any poles and, therefore, the integral around the closed loop is zero. This means that the desired integral equals the sum of the half-residues at $z=z_1$ and $z=z_2$. The residues at $z_{1,2}$ are

$$\left.\frac{1}{(z+1)(z^2-2\mathrm{i}z-2)}\right|_{z=z_1}=\frac{1}{2(1-2\mathrm{i}-2)}=-\frac{1}{2(1+2\mathrm{i})}=-\frac{1-2\mathrm{i}}{2(1+4)}=-0.1+0.2\mathrm{i},$$

$$\left.\frac{1}{(z-1)(z^2-2\mathrm{i}z-2)}\right|_{z=z_2}=\frac{1}{-2(1+2\mathrm{i}-2)}=\frac{1}{2(1-2\mathrm{i})}=\frac{1+2\mathrm{i}}{2(1+4)}=0.1+0.2\mathrm{i}.$$

The sum of the residues at $z_1$ and $z_2$ is thus seen to be $0.4\mathrm{i}$. This must be multiplied by $-\mathrm{i}\pi$, where the minus sign accounts for the counterclockwise direction of rotation around the small semi-circles in Fig.(a). The desired integral is thus equal to $0.4\pi$.

In Fig.(b) the closed contour contains the poles at $z_3$ and $z_4$. The residues at these poles are

$$\left.\frac{1}{(z^2-1)(z+1-\mathrm{i})}\right|_{z=z_3}=\frac{1}{[(1+\mathrm{i})^2-1][(1+\mathrm{i})+1-\mathrm{i}]}=\frac{1}{2(-1+2\mathrm{i})}=-\frac{1+2\mathrm{i}}{2(1+4)}=-0.1-0.2\mathrm{i},$$

$$\left.\frac{1}{(z^2-1)(z-1-\mathrm{i})}\right|_{z=z_4}=\frac{1}{[(-1+\mathrm{i})^2-1][(-1+\mathrm{i})-1-\mathrm{i}]}=\frac{1}{2(1+2\mathrm{i})}=\frac{1-2\mathrm{i}}{2(1+4)}=0.1-0.2\mathrm{i}.$$

The sum of the residues at $z_3$ and $z_4$ is, therefore, $-0.4\mathrm{i}$, which, upon multiplying with $2\pi\mathrm{i}$, yields $0.8\pi$. To this we must now add the sum of the half-residues at $z_1$ and $z_2$. This will be the same as the result obtained in the previous case except for a change of sign — because the direction of travel around the small semi-circles in Fig.(b) is clockwise. Therefore, the desired integral is given by $0.8\pi-0.4\pi=0.4\pi$, as before.