

**Problem 2)** Drawing the diameter  $BB'$  divides each of the angles  $\widehat{ABC}$  and  $\widehat{AOC}$  into two angles. On the left-hand-side of the circle we now have  $\beta = \widehat{ABO}$  and  $\gamma = \widehat{AOB'}$ , for which we are going to prove that  $\beta = \gamma/2$ . The same line of reasoning then applies to the remaining angles on the right-hand-side, namely,  $\widehat{OBC}$  and  $\widehat{B'OC}$ .

The angle  $\gamma$  is the external angle of the  $AOB$  triangle which is supplementary to  $\widehat{AOB}$  (that is, they add up to  $180^\circ$ ). Similarly,  $\alpha + \beta$  is supplementary to  $\widehat{AOB}$ . Therefore,  $\alpha + \beta = \gamma$ . However, the triangle  $AOB$  is isosceles, because  $AO = BO = R$ . Therefore,  $\alpha = \beta$ . Consequently,  $\beta = \gamma/2$ , which completes the proof.

