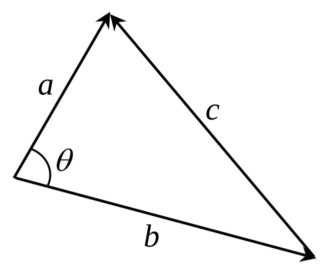
Opti 503A **Midterm Exam** (2/27/2013) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

Problem 1) Consider an arbitrary triangle with sides a, b, and c, as shown.



2 pts a) Treating each side of the triangle as an ordinary vector in 3-dimensional Euclidean space, one may write $\vec{\boldsymbol{c}} = \vec{\boldsymbol{a}} - \vec{\boldsymbol{b}}$. Using elementary vector algebra, derive an expression for the length c in terms of a, b, and the angle θ between a and b.

5 pts b) According to the so-called Heron's theorem, the area A and the half-perimeter s of the triangle are related as follows:

$$A = \sqrt{s(s-a)(s-b)(s-c)}.$$

Here $s = ½(a+b+c)$. Prove Heron's theorem using the result obtained in part (a) and the fact that $A = ½ab\sin\theta$.

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5 pts **Problem 2**) Show that $\displaystyle\sum_{k=0}^{n} \frac{1}{k+1}\binom{n}{k} = \frac{2^{n+1}-1}{n+1}$.

Hint: Integrate $(x+1)^n$ with respect to x from 0 to 1.

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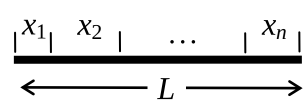
5 pts **Problem 3**) A straight stick of length L is to be cut into n pieces of lengths $x_1, x_2, \ldots, x_n$. The total length of the various pieces must obviously add up to L, that is, $x_1 + x_2 + \cdots + x_n = L$. We would like to devise a strategy for cutting the stick in such a way as to yield the maximum value for the product $x_1 x_2 \cdots x_n$. Use proof by induction to show that $x_1 x_2 \cdots x_n$ is maximized when all the pieces are of equal length, that is, $x_1 = x_2 = \cdots = x_n = L/n$.



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5 pts **Problem 4**) Solve the preceding problem (Problem 3) using the method of Lagrange multipliers.

Problem 5) Use the Cauchy-Riemann conditions to determine the domain of analyticity of each of the following functions:

4 pts a) $f_1(z) = \cos z$;

4 pts b) $f_2(z) = \dfrac{1}{1+\exp(z)}$.
