Part 9
Phase Shifting Interferometry

- Classical Interferogram Analysis
- Phase Shifting Advantages
- Phase Shifters
- Algorithms
- Removing Phase Ambiguities

Classical Analysis of Interferograms

Surface Error = \( \frac{\lambda}{2} \frac{\Delta}{S} \)

- Classical Analysis
  - Measure positions of fringe centers.
  - Deviations from straightness and equal spacing gives aberration.
Computer Analysis of Interferograms

Largest Problem
Getting interferogram data into computer

Solutions
- Graphics Tablet
- Scanner
- CCD Camera
- Phase-Shifting Interferometry

Advantages of Phase-Shifting Interferometry
- High measurement accuracy (>1/1000 fringe, fringe following only 1/10 fringe)
- Rapid measurement
- Good results with low contrast fringes
- Results independent of intensity variations across pupil
- Phase obtained at fixed grid of points
- Easy to use with large solid-state detector arrays
Phase-Shifting Interferometry

1) MODULATE PHASE
2) RECORD MIN 3 FRAMES
3) CALCULATE OPD

OPD = \frac{\lambda}{2\pi} \tan^{-1} \left[ \frac{C - B}{A - B} \right]

Phase Shifting - Moving Mirror

Move \lambda/8

\pi/2 Phase Shift

PZT Pushing Mirror
Phase Shifting - Diffraction Grating

Diffraction Grating +1 Order $\pi/2$ Phase Shift
Move 1/4 Period

Phase Shifting - Bragg Cell

Bragg Cell $f_0$ $f_0 + f$ +1 Order $f_0$ 0 Order Frequency $f$

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Phase Shifting - Tilted Glass Plate

\[ \frac{\pi}{2} \]

Phase Shift

Phase Shifting - Rotating Half-Wave Plate

Circularly Polarized Light

Half-Wave Plate

45° Rotation

\[ \frac{\pi}{2} \]

Phase Shift
Four Step Method

\[ I(x,y) = I_0 + I'_0 \cos[\phi(x,y) + \phi(t)] \]

\[ I_1(x,y) = I_0 + I'_0 \cos \phi(x,y) \quad \phi(t) = 0 \]
\[ I_2(x,y) = I_0 - I'_0 \sin \phi(x,y) \quad \phi(t) = \pi/2 \]
\[ I_3(x,y) = I_0 - I'_0 \cos \phi(x,y) \quad \phi(t) = \pi \]
\[ I_4(x,y) = I_0 + I'_0 \sin \phi(x,y) \quad \phi(t) = 3\pi/2 \]

\[ \tan[\phi(x,y)] = \frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)} \]

Relationship between Phase and Height

\[ \phi(x, y) = \tan^{-1}\left[ \frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)} \right] \]

\[ \text{Height Error}(x, y) = \frac{\lambda}{4\pi} \phi(x, y) \]
Phase-Measurement Algorithms

Three Measurements
\[ \phi = \tan^{-1}\left( \frac{I_3 - I_2}{I_1 - I_2} \right) \]

Four Measurements
\[ \phi = \tan^{-1}\left( \frac{I_4 - I_2}{I_1 - I_3} \right) \]

Hariharan
Five Measurements
\[ \phi = \tan^{-1}\left[ \frac{2(I_2 - I_4)}{I_3 - I_5 - I_1} \right] \]

Carré Equation
\[ \phi = \tan^{-1}\left[ \frac{\sqrt{3}(I_2 - I_3) - (I_1 - I_4)(I_2 - I_1) - (I_1 - I_4)}{(I_2 + I_3) - (I_1 + I_4)} \right] \]

Phase-Measurement Algorithm for N Intensity Measurements

N Measurements
\[ \phi = -\tan^{-1}\left[ \frac{\sum_{i=1}^{N} I_i \sin \alpha_i}{\sum_{i=1}^{N} I_i \cos \alpha_i} \right] \]

\[ \alpha_i = \frac{2 \pi i}{N} \quad \text{for } i = 1, \ldots, N \]

Technique is also known as synchronous detection

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Phase-Stepping Phase Measurement

Integrated-Bucket Phase Measurement
Integrating-Bucket and Phase-Stepping Interferometry

Measured irradiance given by

\[ I_i = \frac{1}{\Delta} \int_{\alpha_i - \Delta/2}^{\alpha_i + \Delta/2} I_o \left\{ 1 + \gamma_o \cos[\phi + \alpha_i(t)] \right\} d\alpha(t) \]

\[ = I_o \left\{ 1 + \gamma_o \text{sinc} \left[ \frac{\Delta}{2} \right] \cos[\phi + \alpha_i] \right\} \]

Integrating-Bucket \( \Delta = \alpha \)
Phase-Stepping \( \Delta = 0 \)

Phase Ambiguities

2\( \pi \) Phase Steps
Phase
Phase
BEFORE

Phase Steps Removed
Phase
Phase
AFTER

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Removing Phase Ambiguities

- Arctan Mod $2\pi$ (Mod 1 wave)
- Require adjacent pixels less than $\pi$ difference

$\text{(1/2 wave OPD)}$

- Trace path
- When phase jumps by $>\pi$
  Add or subtract $N2\pi$

Adjust so $<\pi$

Phase-Shifting Interferometer
References - 1


References - 2

References - 3

- S. Tolansky, Multiple-Beam Interferometry of Surfaces and Films (Dover, New York, 1970).