What is an Imaging Spectrometer?

• The objective of an Imaging Spectrometer is the acquisition of a set of registered, spectrally contiguous images of a scene’s spatial radiance distribution.

• Future trend: choice of spectral bands tailored to a particular task; instrument provides immediate answers rather than more data.

– 512 by 512 by 200 bands = 110 megabytes of data / information = ?
  -How fast can data be acquired @ a reasonable SNR?
• Multispectral imaging – driven by then-available technology (i.e. filters with 10% bandwidth of $\lambda$ center)

• Hyperspectral imaging
  – large quantities of data from which information must be extracted
  – hyperspectral represents “discovery” mode
  – 1% bandwidth

• What lies beyond?
  – ultraspectral: (0.1%) used for imaging of effluents which exhibit narrow spectral features due to molecular rotational degrees of freedom
  – compact, lighter systems capable of “on-board” processing—electronic or optical; extraction of answers in real-time
The Data Cube

- Three-dimensional (3D) data set: Two spatial dimensions and a spectral dimension
- Spectral data highly redundant in wavelength
- Data exploitation can be based on spatial cues, pixel spectra (z-profile), or some combination
- Image cube, object cube, hypercube

Multispectral/ Hyperspectral Comparison

http://iris.usc.edu/home/iris/huertas/www/hydice
A continuous sampled spectrum is associated with each pixel of object cube.

Spectral bands must be spatially (x,y) registered.

Single-pixel spectra
GSD = 17 x 17 m
Altitude = 20 km

λ - Wavelength, μm

Impact of New Technologies

- Modern focal-plane arrays (FPAs) coupled with previously developed spectrometric concepts allow spatial and spectral imaging with higher SNR.

- Modern FPAs allow spectral imaging in the visible-to-NIR (0.4-1.0 μm), SWIR (1.0-2.5 μm), MWIR (3–5.5 μm), and LWIR (8–12 μm).

- The continuing trend toward more powerful computers and software makes rapid acquisition of large quantities of data feasible and useful (limited to 75 MHz per channel).

- New techniques for data exploitation and extraction of information allow the user unprecedented insight into object space; anomaly detection, sub-pixel analysis, and statistical methods for removal of environmental effects.
Data acquisition within the data cube is performed differently by different spectrometer configurations.

- Slit-spectrometer (A) collects a “wall” of data: pushbroom scanning (like HYDICE or COIS) allows acquisition of a complete data cube.
- Filter-wheel camera (B) collects “plates”. Different filters must be “dialed in” to acquire data cube.
- Whiskbroom (C) requires faster scanning for data cube acquisition (AVIRIS).
Forms of Spatial Scanning

- In remote sensing especially, the motion of the observing platform can provide the scanning motion necessary for imaging.

- AVIRIS executes a whiskbroom form of scanning

- HYDICE executes a pushbroom scan

- Filtered staring array: no spatial scanning

www.ltid.inpe.br/html/pub/docs/html/aboutAV.htm
Satellite Ground Track

Image Acquisition Modes

http://www.ba.dlr.de/NE-WS/ws5/sensor/sensors.html
Spectral images may not be aligned due to platform movement from frame to frame.
Pushbroom Scanning

Note the dynamic range requirements on the sensor

Any resolution element (GSD) will have a valid spectrum, but adjacent spatial element registration in direction of scan may be in error.

http://speclab.cr.usgs.gov/index.html
Whiskbroom Scanning

Spectrally correct for any IFOV, but may not be spatially registered in $x$ and $y$ directions.

http://aviris.jpl.nasa.gov/
Wedge Spectrometer

Hardware simpler but both

http://www.ba.dlr.de/NE-WS/ws5/sensor/sensors.html
Discrete Object Representation

- Volume elements (voxels); $\Delta x \cdot \Delta y \cdot \Delta \lambda$
- Voxel basis forms an approximation of the object
  - object is continuous but sensor collects discrete samples
- Number of voxels depends on application requirements and imaging spectrometer hardware
- Lots of data: millions of voxels in a data cube
- This is true for all spectral imaging technologies
- There is no continuous measurement technique
Whiskbroom Scanning

- Cross-track scan motion (mirror)
- Dispersion of IFOV: a linear array of detectors in place of exit slit; multidetector advantage
- Disadvantage: shorter dwell time and thus reduced signal-to-noise ratio (SNR); equivalently, higher bandwidth needed – $\Delta f = 1/(2t_{\text{dwell}})$
  
  - Advantage: Fewer detector elements to calibrate
  - Bow-tie effect: away from nadir, path to ground increases and so does the IFOV footprint

Increasing ground sampling distance
Pushbroom Scanning

- Scan motion along track due to platform motion, no other scanning required
- Requires a staring (2D) focal-plane array
- One-dimensional spatial imaging, FOV defined by the entrance slit length
- Dispersed radiance from each IFOV is detected along a row of the FPA
- Spatial multidetector advantage relative to whiskbroom: dwell time increases by $N = \text{FOV/IFOV}$, SNR increases by $\sqrt{N}$
- Disadvantages:
  - Many more detectors to calibrate
  - Subject to the “bow-tie” effect at edges of FOV
  - Smile and spectral keystone distortion
Slit Curvature and Keystone Distortion

• Smile: slit curvature
  - Characterize with monochromatic uniform fill of entrance slit
  - Pincushion distortion; magnification is function of field angle

Forward to slide 38-39

• Spectral keystone
  - Characterize with white-light pinhole moving along entrance slit
  - Measure centroid in each column
  - Magnification is function of wavelength (Transverse Chromatic)

http://www.vanwalree.com/optics/chromatic.html

• Should be sub-pixel, 1/10 of a pixel, to avoid correction by post-processing

You can virtually build your own pushbroom imaging spectrometer: need fore-optics, a monochromator, and a CCD camera.

Dispersion & Spectral Measurement Techniques

- Prisms
- Gratings
  - Blazed
  - Echelle
- Hadamard
- Fourier Transform
Disperser: Prism

• Advantage: High and constant efficiency over the material’s transmittance range
• Disadvantage: limited dispersion; larger for shorter wavelengths and non-linear

Angular dispersion \( \frac{d\theta}{d\lambda} = \left( \frac{b}{t} \right) \frac{dn}{d\lambda} \)

Most optical glasses obey \( n(\lambda) = A + \frac{B}{\lambda^2} \)

Resulting angular dispersion is greater for shorter wavelengths and non-linear \( \frac{d\theta}{d\lambda} = -\frac{2}{t} \frac{b}{\lambda^3} \)

Herschel used prism to discover IR
Achromatized Prism

- Multi-glass prism designed to yield linear dispersion
- Simple least squares design method treats prisms as thin wedges
- Each glass characterized by index of refraction, Abbe number, and relative partial dispersion
- Specify:
  - center wavelength deviation
  - total deviation
  - center and long wavelength angle
- 3 glasses satisfy constraints
Disperser: Grating (Geometry)

- Disperser: Grating
- Normal
- $\gamma$
- $\alpha$
- $\beta$
- $\sigma$
- $N = 11$
- $m$ : order
- $N$ : Number of lines in beam
- $\sigma$ : line spacing
- $\alpha$ : angle of incidence
- $\beta$ : angle of diffraction
- $\gamma$ : out-of-plane angle

Grating Normal
Disperser: Grating

• Grating equation (for reflection grating): \( m\lambda = \sigma (\sin \alpha + \sin \beta) \)
• The more complete grating equation: \( m\lambda = \sigma \cos \gamma (\sin \alpha + \sin \beta) \)
• Resolving power: \( \frac{\lambda}{\delta \lambda} = mN \)
• Advantage: achieve desired dispersion through groove spacing
• Disadvantage:
  – Efficiency changes over a wide spectral range; the variation of efficiency with wavelength can be “centered” on the desired order by “blazing”
  – Polarization sensitivity as \( \sigma \approx \lambda \)
• Order overlap, free spectral range: \( \Delta \lambda = \frac{\lambda}{m} \)
• Use filter or cross-disperser to use overlapped orders

Back to slide 32
Prism vs. Grating Dispersion

- Prism: Material dependent wavelength
- Grating: Any spectral region

- Prism Angular Dispersion (rad/μm)
- Fused silica
- $b/t = 0.5$

- Grating Angular Dispersion (rad/μm)
- Three periods: 5, 10, 15 μm

Note the superior linearity of the gratings vs. the prism disperser.
Echelle Cross-disperser Spectrometer

- Echelle is a coarse grating (~3 lines/mm) operating in very high diffraction orders
- The free-spectral range is very small
- Dispersed orders overlap and a cross-disperser is needed to separate them
- Cross-disperser can be a prism or another grating
- Nominally: 2-D array, non-imaging, high spectral resolution instrument

Ref: http://www.llnl.gov
Lesson 16-26

Pictorial Explanation of Echelle Cross
Disperser Effect

- Grating at high order cause overlap

- 2nd grating or prism to produce vertical shift in spectra

- Now any given wavelength is isolated into orders which are spread over 2-D array

- Horizontal row of detectors shows repeated wavelength from different orders
• Boxed regions indicate same spectral regions at different orders

http://www.mmto.org
Linear Algebra in Spectrometry

- Originally makes its appearance in Hadamard spectrometers and Fourier transform
- Offers multiplex advantage
- Fellgett’s advantage; opening up the exit slit (multiple exit slits)
- Multiplexing in spectral variable ($\lambda$) only

$$g = HE$$

- Each element of $g$ is a multiplexed measurement, $E$ is the spectral irradiance at the exit-slit plane, and $H$ describes the multiplexing

$$E = [E(\Delta\lambda_1), E(\Delta\lambda_2), E(\Delta\lambda_3), \ldots, E(\Delta\lambda_n)]^T$$

- Next: a few examples...
Lesson 16-29

Prism Spectrometer

**PRISM:** One wavelength \(\rightarrow\) One propagation angle

\[
\begin{bmatrix}
E(\Delta\lambda_1) \\
E(\Delta\lambda_2) \\
E(\Delta\lambda_3) \\
E(\Delta\lambda_4)
\end{bmatrix}
\]

Trivial case: \(H\) is diagonal

\[
\begin{bmatrix}
g_1 \\
g_2 \\
g_3 \\
g_4
\end{bmatrix} =
\begin{bmatrix}
h_{11} & 0 & 0 & 0 \\
0 & h_{22} & 0 & 0 \\
0 & 0 & h_{33} & 0 \\
0 & 0 & 0 & h_{44}
\end{bmatrix}
\begin{bmatrix}
E(\Delta\lambda_1) \\
E(\Delta\lambda_2) \\
E(\Delta\lambda_3) \\
E(\Delta\lambda_4)
\end{bmatrix}
\]

Approximation: each detector sees a finite bandwidth
Disperser: Grating

- Free spectral range
- Multiple orders & overlap
- Not just one dispersion
  - choose order to change dispersion
Hadamard Spectrometer

- Baseline instrument: a single-entrance-slit / single-exit slit monochromator with one detector
- Hadamard: multiple exit slits combined onto one detector
- At each measurement, the detector sees a linear combination of spectral signals
  - exciting possibility: if we can adjust which slits to open and which slits to close, using grayscale transmittance, then we generate an adjustable filter
Hadamard Spectrometer II

- Mask with $2N-1$ slots for encoding the spectral distribution:

Motion of mask, cyclic

- Only 7 slots are used at a time
- The $H$-matrix is cyclic; after each measurement, translate mask by one slot

$g_1 = E(\Delta \lambda_1) + E(\Delta \lambda_2) + E(\Delta \lambda_3) + E(\Delta \lambda_5) \ldots$
**Hadamard: Inverting the Measurements**

Q: How do you go from \( g \) to \( E(\lambda) \)? \( E(\lambda) = H^{-1}g \)

\[
\begin{bmatrix}
E(\Delta\lambda_1) \\
E(\Delta\lambda_2) \\
E(\Delta\lambda_3) \\
E(\Delta\lambda_4) \\
E(\Delta\lambda_5) \\
E(\Delta\lambda_6) \\
E(\Delta\lambda_7)
\end{bmatrix} =
\begin{bmatrix}
0.25 & -0.25 & -0.25 & 0.25 & -0.25 & 0.25 & 0.25 \\
0.25 & 0.25 & -0.25 & -0.25 & 0.25 & -0.25 & 0.25 \\
0.25 & 0.25 & 0.25 & -0.25 & -0.25 & 0.25 & -0.25 \\
-0.25 & 0.25 & 0.25 & 0.25 & -0.25 & -0.25 & 0.25 \\
0.25 & -0.25 & 0.25 & 0.25 & 0.25 & -0.25 & -0.25 \\
-0.25 & 0.25 & -0.25 & 0.25 & 0.25 & 0.25 & -0.25 \\
-0.25 & -0.25 & 0.25 & -0.25 & 0.25 & 0.25 & 0.25
\end{bmatrix}
\begin{bmatrix}
g_1 \\
g_2 \\
g_3 \\
g_4 \\
g_5 \\
g_6 \\
g_7
\end{bmatrix}
\]

- **Key concept:** be able to invert the H-matrix to get spectral data
- **7 independent equations, 7 unknowns**
Hadamard

- Each measurement = sum of selected spectral channels

Multiplexing in Wavelength

- Noise: signal-independent
- Result: improve SNR of each measurement
Fourier-Transform Spectrometer (FTS)

- Michelson interferometer
- No dispersion, but source spectrum can still be extracted
- Resolution of an interferometer varies inversely as the maximum OPD
FTS

- Weighted sums again like a Hadamard
- Choice of weights is determined by the interference of light in a Michelson
- Weights are non-negative
- Example of transmission masks to convert a Hadamard to a Fourier transform spectrometer

Increasing Path Difference

Lesson 16-36
FTS Spectral Resolution

- The resolution of an FTS is related to the physical distance covered by the scanning mirror:

\[ \Delta v = \frac{1}{2 \Delta x} \text{[cm}^{-1}] \]

\( \Delta x = \text{mirror travel} \)

- Resolution is constant in wavenumber
- Typical resolutions are 0.5, 1, 2, 4, 8, 16 cm\(^{-1} \)
Scan Time

- Scan time may be approximated by:

\[ T = \frac{\Delta x}{V_m} = \frac{1}{2\Delta v_m} \]

\[ V_m = \text{scan mirror velocity (0.633 cm/s)} \]

<table>
<thead>
<tr>
<th>( \Delta v (\text{cm}^{-1}) )</th>
<th>( T ) (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>0.05</td>
</tr>
<tr>
<td>8</td>
<td>0.1</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
</tr>
<tr>
<td>1</td>
<td>0.8</td>
</tr>
</tbody>
</table>
• A non-imaging Fourier-transform spectrometer measures irradiance as a function of path-difference

• Maximum oscillations occur near zero-path-difference (ZPD)

Reference:
http://www.cfht.hawaii.edu/Instruments/Spectroscopy/FTS
Space Sensor Design

- Orbital Issues
- Spectral Requirements and Resolution
- Spatial Resolution and Swath Width
- Revisit Time
- Signal expected
Sensor Specifications

• Spectral Resolution
  – Bandwidth: 0.5 to 2.5 μm
  – 10 nm sampling; 20 nm resolution

• Spatial Requirements
  – 5 m GSD
  – 5 km swath

• Orbital requirements (e.g. www.stk.com)*
  – 7 day revisit time
  – 5 year lifetime; this sets a limit on minimum altitude (420 km)

• Sensitivity Requirements
  – SNR > 25 for albedo (ρ) of 0.2

* satellite tool kit
Orbital Constraints

• Satellite velocity

\[ V_c \ [\text{km/s}] = 7.9054 - 6.197 \times 10^{-4} \ H [\text{km}] \]

• Ground Speed (Typically 7 to 8 km/sec)

\[ V_g = \frac{R_e}{R_e + H} V_c \]

Example

\( R_e = \) Earth’s radius, 6378 km
\( H = \) Satellite altitude, 420 km
\( V_g = 7.17 \ \text{km/sec} \)
Optical System Flowdown

• Assume Diffraction Limited ($\lambda = 2.5 \, \mu m$)

$$\frac{2.44\lambda}{D_o} = \frac{5m}{420km}$$

$D_o$: Optics Diameter = 0.51 m

• Detector Size (off-the-shelf): 19 $\mu m$ x 19 $\mu m$

$$\sqrt{A_d} = \frac{2.44\lambda f}{D_o}$$

f = 1.6 m or F/# = 3.1
Optical System Flowdown

- IFOV is 11.87 μrad,

\[
\text{IFOV} = \frac{19\mu\text{m}}{1.6 \text{ m}}
\]

- Full FOV is \(1000(\text{IFOV}) = 11.87\) mrad
Optical Specifications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swath Width</td>
<td>5 km</td>
</tr>
<tr>
<td>Ground Sampling Distance (GSD)</td>
<td>5 m</td>
</tr>
<tr>
<td>Aperture Diameter</td>
<td>0.51 m</td>
</tr>
<tr>
<td>Telescope Type</td>
<td>Cassegrain</td>
</tr>
<tr>
<td>Obscuration</td>
<td>&lt;30 %</td>
</tr>
<tr>
<td>Effective Focal Length</td>
<td>1.6 m</td>
</tr>
<tr>
<td>IFOV</td>
<td>11.87 μrad</td>
</tr>
</tbody>
</table>
Spectral Separation Technique

- **Prism Type**
  - Achromatize for approximately linear dispersion
  - 10 nm sampling (Appendix C)

- **Two Spectral Regions Due to Responsivity of Detectors**
  - Visible (VNIR): 0.5 to 1.0 μm
  - Short wave infrared (SWIR): 1.0 to 2.5 μm

- **Channels: 200**
  - VNIR = 50
  - SWIR = 150
## Detector Specifications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detector Material</td>
<td>Si and HgCdTe</td>
</tr>
<tr>
<td>$D^* [\text{cm-Hz}^{1/2}/\text{watt}]$</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>Number of Pixels</td>
<td>1000</td>
</tr>
<tr>
<td>Pixel size</td>
<td>19 microns</td>
</tr>
<tr>
<td>Integration Time</td>
<td>333 $\mu$sec</td>
</tr>
<tr>
<td>Read Noise</td>
<td>70 e$^{-}$</td>
</tr>
<tr>
<td>A/D Converter</td>
<td>12 bits</td>
</tr>
</tbody>
</table>
Sensitivity Analysis

- Radiometry
- Figure of Merit of Spectrometer (NESR)
- Signal to Noise for Reflectivity ($\rho$) of 0.2
• The spectral power collected by an instrument is given by

\[ \Phi_d = L A \Omega \]

• Assume the source is Lambertian; it radiates or reflects equally into a hemisphere. In the case of reflection:

\[ L = \rho E/\pi \left( \frac{\text{Watt}}{m^2 \cdot \text{sr} \cdot \mu\text{m}} \right) \]

• The throughput (A\(\Omega\) product)
• \(A_s\): source area, \(\Omega\) defined by collection optics in object space
• \(A_e\): entrance slit, \(\Omega\) defined by object’s subtends
- What is the radiance which results in an SNR = 1?

- NESR(\(\lambda\)) as a function of spectrometer parameters

\[
\text{NESR}(\lambda) = \frac{1}{\sqrt{2t_{\text{int}}}} \cdot \frac{1}{D^*} \cdot \frac{1}{\Delta \lambda} \cdot \frac{1}{\tau(\lambda)} \cdot \left[ \frac{4F^2}{\pi \sqrt{A_d}} \right]
\]

- Assumptions:
  - Scene varies little within IFOV and so can be treated as an extended source
  - Dwell time: \(t_{\text{dwell}} = t_{\text{int}}\)

- Figure of Merit often quoted.
The scene radiance, $L_{s\lambda}$, is the NESR when $SNR = 1$.

Substitution of the radiant power-expression ($\Phi_d$) into the specific detectivity ($D^*$) equation yields the NESR.

Example: The scene radiance is the product of solar irradiance and scene BRDF. Assuming a Lambertian reflector, NESR may be converted into noise equivalent spectral reflectance, $NES\rho$.
Lesson 16-52

**NESR Derivation**

- Begin with the expression for specific detectivity,

\[ D^* = \frac{\sqrt{A_d \cdot \Delta f}}{\text{NEP}} = \frac{\sqrt{A_d \cdot \Delta f}}{\Phi_d} \text{SNR} \]

- The detector bandwidth is related to dwell time and integration time:

\[ \Delta f = \frac{1}{2t_{\text{dwell}}} = \frac{1}{2t_{\text{int}}} \]

- Now assume an extended source. The power on the detector is:

\[ \Phi_d = \tau(\lambda) L_{s\lambda} \Delta \lambda A_d \]

\[ \Omega = \frac{\tau(\lambda) L_{s\lambda} \Delta \lambda A_d \pi}{4(F^\#)^2} \]

\[ \Omega = \frac{\pi}{4F^2 + 1} \]

Definition: Extended source fills the FOV of a detector element.
\[ D^* = \frac{\sqrt{A_d \Delta f}}{\Phi_d} \cdot \frac{S}{N} \]

\[ \Phi = \frac{\tau L_e A_d \pi}{4(F/\#)^2} = \frac{\tau L_e \Delta \lambda A_d \pi}{4(F/\#)^2} \]

\[ D^* = \frac{\sqrt{A_d \Delta f}}{\tau L_e \Delta \lambda A_d \pi} \cdot \frac{4(F/\#)^2}{\sqrt{2\tau_{int} \cdot \tau \Delta \lambda D^* \pi \sqrt{A_d}}} \cdot \frac{S}{N} \]

spectral radiance for \( \frac{S}{N} = 1 \)
Lesson 16-54

Noise-Equivalent Spectral Radiance

• What is the radiance which results in a $SNR = 1$?
• $NESR(\lambda)$ as a function of spectrometer parameters

$$NESR(\lambda) = \frac{1}{\sqrt{2t_{int}}} \cdot \frac{1}{D^*} \cdot \frac{1}{\Delta \lambda} \cdot \frac{1}{\tau} \left[ \frac{4(F/#)^2}{\pi \sqrt{A_d}} \right]$$

• Assumptions: scene varies little within IFOV and so can be treated as an extended source. The dwell time $t_{dwell} = t_{int}$.
• Figure of Merit often quoted
• Dependence of $NESR$ on GSD:

$$NESR = \frac{1}{\sqrt{2\tau_{\text{int}}}} \cdot \frac{1}{D^*} \cdot \frac{1}{\Delta \lambda \tau(\lambda)} \left[ \frac{4(F/\#)^2}{\pi \sqrt{A_d}} \right]$$

$$t_{\text{int}} = \frac{GSD}{v_g} \Rightarrow F/\# \equiv \frac{f}{D_0}$$

$$NESR = \frac{1}{\sqrt{2}} \cdot \frac{GSD}{v_g} \cdot \frac{1}{D^* \Delta \lambda \tau(\lambda)} \left[ \frac{4}{\pi} \cdot \frac{f^2}{D_0^2 \sqrt{A_d}} \right]$$

$$\frac{f}{\sqrt{A_d}} = \frac{R}{GSD} \Rightarrow A_c = \frac{\pi D_0^2}{4}$$

$$NESR = \frac{\sqrt{v_g}}{\sqrt{2} D^* \Delta \lambda \tau A_C} \cdot \frac{1}{\sqrt{GSD}} \left[ \frac{f^2}{\sqrt{A_d}} \right]$$

• Time-dependent noise assumed,
• Maximize GSD to minimize $NESR$
Signal and GSD Relations

Signal = $L_{\lambda} \cdot \Delta \lambda \cdot \frac{A_S A_C}{H^2} \cdot \tau \cdot t_{\text{int}}$

- $L_{\lambda}$ - source radiance
- $\tau$ - transmission
- $t_{\text{int}}$ - integration time
- **Noise possibilities:**
  - fixed noise level, i.e., READ NOISE, independent of time and signal
  - photon noise or time dependent noise

- **Recall that**
  \[ t_{\text{int}} = \frac{\text{GSD}}{v_g} \]

- **GSD** - Ground Sampling Distance

Lesson 16-56
Signal-to-Noise Calculation

- Recall: \[ D^* = \frac{\sqrt{A_d \cdot \Delta f}}{\Phi_d} \text{SNR} \]

- Rearranging NESR expression:

\[
\text{SNR} = \frac{\pi}{4} \left( \frac{D_o}{f} \right) \frac{D_o \sqrt{A_d}}{f} \cdot \frac{\tau_a \tau_o \rho}{\Omega_s} \Omega_s \int d\lambda \ L_s D^* \]

- \( \tau_a = 0.7 \) (atm. transmission)
- \( \tau_o = 0.32 \) (optics transmission)
- \( \Omega_s = \pi \sin^2(0.25^\circ) \) (solar solid angle)
- \( L_s \) = sun’s radiance
Integration Time Calculation

• Dwell Time

\[ t_{\text{int}} = \frac{5 \text{m}}{7.17 \text{ km/sec}} = 700 \mu\text{s} \]

To lessen the smear we assume only 333 \( \mu\text{s} \) integration time, \( t_{\text{int}} \)

• Lambertian Surface

\[ \text{BRDF} = \frac{\rho}{\pi} \]
Solar Radiance Calculations

- **1 micron value**

\[ \int_{10 \text{nm}}^{s} L_s(5900\text{K}, \lambda) d\lambda = 11.3 \text{ Watt/cm}^2 \text{ sr} \]

- **2.5 micron value**

\[ \int_{10 \text{ nm}}^{s} L_s(5900\text{K}, \lambda) d\lambda = 0.73 \text{ Watt/cm}^2 \text{ sr} \]
Signal-to-Noise at Extreme Wavelengths

• 1 micron

$$\text{SNR} = \frac{\pi}{4} \left( \frac{51}{160} \right)^2 \frac{(19 \times 10^{-4}) (0.7)^2 (0.32) (0.2)}{\pi \sqrt{1/2(333 \times 10^{-6})}} (5.98 \times 10^{-5})(10^{12}) 11.3$$

$$= 26$$

• 2.5 micron

$$\text{SNR} = \frac{\pi}{4} \left( \frac{51}{160} \right)^2 \frac{(19 \times 10^{-4}) (0.7)^2 (0.32) (0.2)}{\pi \sqrt{1/2(333 \times 10^{-6})}} (5.98 \times 10^{-5})(10^{12}) 0.73$$

$$= 2$$

• Next question: What specifications/design aspects need to be altered to increase the SNR’s?